Experiment MP-2

Electron Diffraction off Graphite

Educational Objective

To study wave-like properties of massive particles (i.e. electrons).

Experimental Objective

To measure the deflection of an electron beam due to diffraction by virtue of its de Broglie wavelength.

Apparatus

One Tel-Atomic CRT tube containing:

1. an electron gun,
2. an evacuated clear glass bulb on which is deposited a luminescent screen, and
3. a target of graphitized carbon.

Voltage and current meters to:

1. monitor the potential on the reversed-bias field, and
2. monitor the current in the high voltage supply; not to exceed 0.2 mA.

Method

By varying the potential in the electron gun, one can vary the momentum (i.e. the de Broglie wavelength) of the electrons.

Secondly, one can adjust the intensity of the electron beam by varying the reversed-bias potential on the *Cathode can* (see figure 1). This can be done by varying the potential on the 50-Volt supply. Use this adjustment to keep the HV (high voltage) current less than 0.2 mA. There's no reason to have the HV current exceed 0.1 mA.

Two rings are formed on the luminescent screen whose radii are determined by the de Broglie wavelength and the interplanar spacing of the atoms in the target (i.e.graphitized carbon). By plotting the accelerating voltage vs. the diameter of the rings, one can determine the interplanar spacings of the target.

Theory

Connect the tube as shown in Fig. 1, switch on the heater supply (i.e. the 6-Volt supply) and wait one minute for the cathode heater to stabilize. Slowly adjust the HV supply to about 2000 Volts. As the high voltage increase be careful to not allow the central spot on the luminescent surface of the evacuated bulb to become too bright. As you increase the high voltage slowly turn up the reversed-bias by adjust the 50-Volt supply.

Two prominent rings should appear around the central spot (see figure 2). An increase in the high voltage causes a decrease in the ring diameter while a decrease in high voltage results in an increased ring diameter. This is in accord with de Broglie's suggestion that wavelength increases with a decrease in momentum. This experiment demonstrates the wave-like properties of the electron, thus revealing its dual nature.

The de Broglie wavelength of a matter particle is

$$\lambda = \frac{h}{mv} \qquad (1)$$

where h is Planck's constant, m is the mass, and v is the velocity. The velocity can be obtained from the classical expression:

$$eV = \frac{1}{2}mv^2 \qquad (2)$$

where V is the potential of the high voltage supply and e is the charge of the electron. Substituting equation (2) into equation (1) we find:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{1.23\ nm}{\sqrt{V}} \qquad (3)$$

A calculation using de Broglie's equations shows that electrons accelerated through a potential difference of 4 kV have a wavelength of about 0.02 nm. Interference and diffraction effects, as studied in physical optics, demonstrate the existence of waves, where for a simple ruled grating, the condition of diffraction is

$$\lambda = d \sin \vartheta, \qquad (4)$$

where d is the spacing of the grating and where for small angles sin θ = θ.

The best man-made gratings are ruled at 2,000 lines per mm and with a wavelength of 0.02 nm, the angle θ will be less than one second of arc or only 0.5 mm at 10 m from the grating. If electron diffraction is to be observed in our evacuated tube with a path length of 140 mm, the spacing between *'rulings'* to produce a first order of interference at 14 mm from zero degrees (i.e. sin θ = 0.1), must be 0.2 nm.

In 1912, Prof. Max von Laue had suggested, in connection with X-ray studies, that if fine gratings could not be made by man because of the basic granularity of matter, then perhaps this very granularity might provide a suitable grating. Sir Lawrence Bragg used the cubic system of NaCl (i.e. salt) to calculate interatomic spacings and showed them to be of the right order for X-rays. This salt, like most salts is not suitable for sealing into an evacuated tube; however carbon is vacuum-stable and can be formed in many different ways.

The condition for diffraction for small angles using equation (4) above is

$$\lambda = d\vartheta \qquad (5)$$

where the small angle θ can be calculated from the geometrical relationship of Figure 2 as

$$\vartheta = \frac{D/2}{L} \qquad (6)$$

and so from equation (3) we have

$$D = \left[\frac{1.23(nm)\ 2\ L}{d}\right]\frac{1}{\sqrt{V}} \qquad (7)$$

Using the slope of the lines recorded in the table in figure 3a and equation (7), the interplanar spacing, d, can be calculated.

Procedure

(1) Turn on the ammeters and voltmeters resting next to the power supplies.

(2) Turn on the 6-Volt supply which is connected to the heating element inside the evacuated tube. A dull red-orange glow should appear in the electron gun.

(3) Making sure that the high voltage *coarse* and *fine* knobs are turned down to their respective minimum values, turn on the high voltage power supply and the 50-Volt supply. Increase the high voltage by turning the coarse knob clockwise. **As you do make sure that you control the current in the high voltage supply by increasing the voltage on the 50-Volt reversed-bias supply.**

Protection of the Carbon Target.

The graphitized carbon through which the electron beam is confined to pass is only a few molecular layers in thickness and can be punctured by current overload.

The purpose of the "reversed-bias voltage" is to reduce the likelihood of damage to the target due to accidental user-abuse. The total emitted current passes through the resistor R; increase in the current causes the cathode-can (see figure 1) to become more negatively biassed, so reducing the emitted current.

Practical Precautions.

Current overload causes the target to become overheated and to glow dull-red. It is good practice to inspect the target periodically during an experiment and especially at switch-on when at least one minute should be allowed for the cathode temperature to stabilize before applying the high voltage.

As an additional safeguard, the anode (high voltage) current should be metered and never allowed to exceed 0.2 mA. Larger high voltages can be achieved without exceeding this limit by reducing the heater voltage.

(4) Figure 3a shows the table that needs to be filled with the data from this experiment. In particular the inner and outer diameters of the rings observed on the luminescent screen need to be recorded for different high voltages, labelled V_a.

(5) Figure 3b shows a typical plot of the $1/\sqrt{V}$ vs. the diameter of the two rings (meters). You should plot similar lines using the data you collect in the table in figure 3a.

(6) Figure 4 shows geometrical relationship between the interplanar spacings corresponding to the two different diffraction gratings in graphitized carbon. The two different slopes in figure 3b correspond to the two d values, d_{10} and d_{11}.

d_{10} (inner) = 0.213 nm
d_{11} (outer) = 0.123 nm

Note: Graphical Construction

The convention of proportionality has been inverted for the purposes of this graphical construction in order to facilitate the calculation of d_{11} and d_{10} from the slopes of the respective lines.

The accuracy of these calculations depends on the length of the line and the caliper measurement of the ring diameters.

Note: Measurement of ring diameters

For maximum accuracy the ring diameter should be extrapolated as in figure 5 in order to compensate for both the curvature and the thickness of the glass envelope.

Specification:

FILAMENT VOLTAGE (V_F) ... 6.3 V ac/dc (8.0 V max.)

ANODE VOLTAGE (V_A) ... 2500 - 5000 V dc

ANODE CURRENT (I_A) ... 0.15 mA at 4000 V (0.20 mA max.)

FIGURE 1

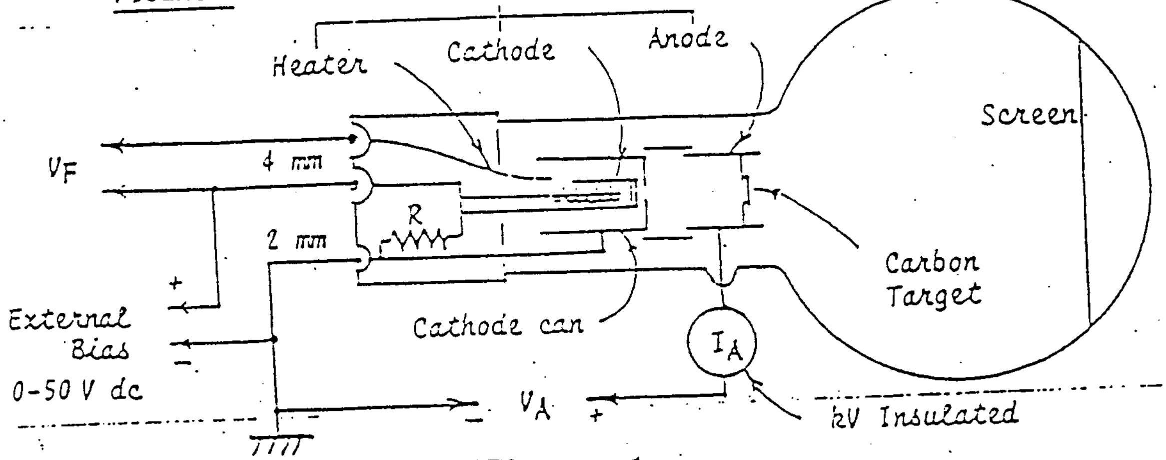

Figure 1

FIGURE 2

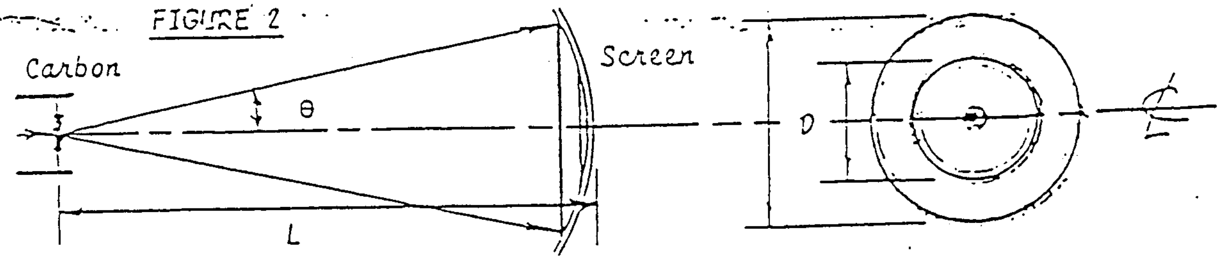

Figure 2

V_a	$V_a^{-1/2}$	D metres	
kV	volts $^{-1/2}$	inner	outer
2.5	0.0200		
3.0	0.0183		
3.5	0.0169		
4.0	0.0158		
4.5	0.0149		
5.0	0.0141		

Figure 3a

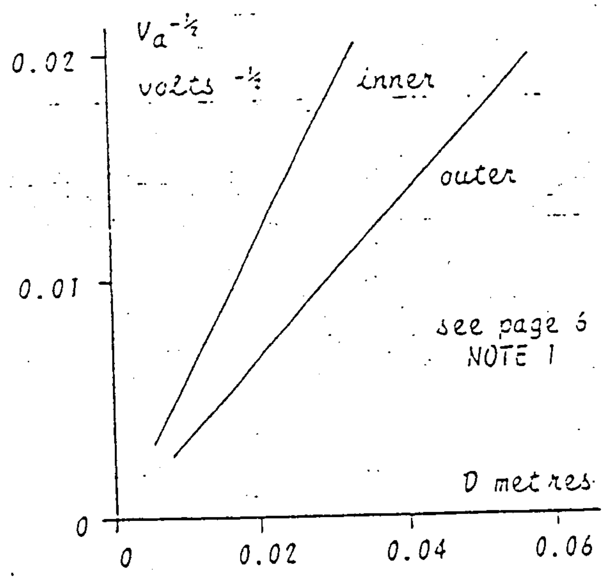

Figure 3b

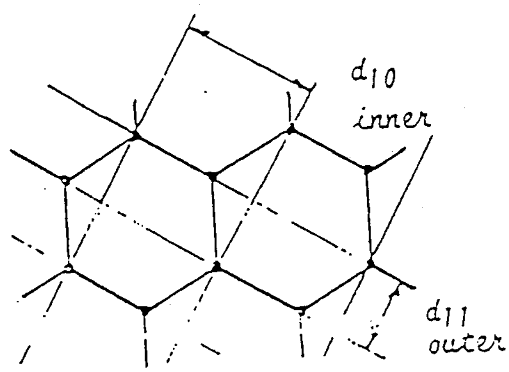

Figure 4

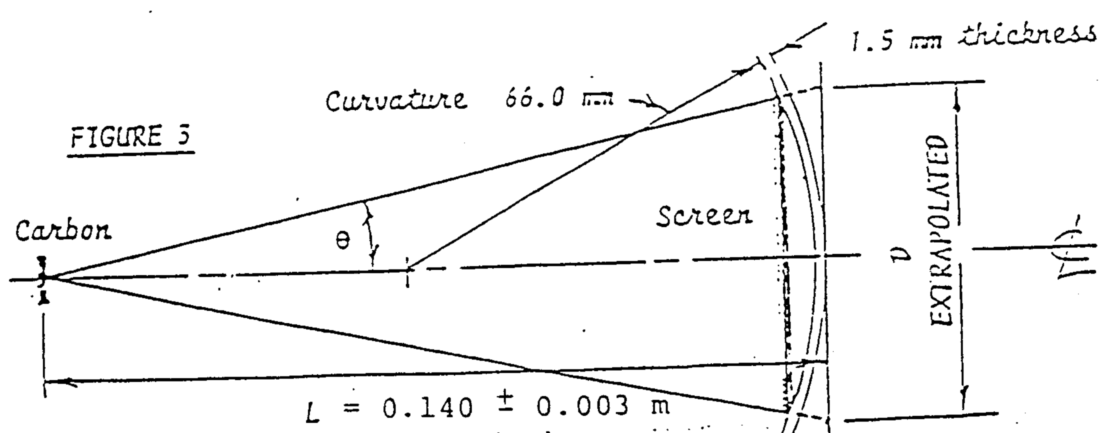

Figure 5

Electron Diffraction Spread 5.0

	A	B	C	D	E
1	Voltage	1/ SQRT(V)	Diameter	W(i)	
2	2500	0.02	0.0337	25000000	
3	3000	0.01825742	0.0302	43200000	
4	3500	0.01690309	0.0281	68600000	
5	4000	0.01581139	0.0261	102400000	
6	4500	0.01490712	0.0251	145800000	
7	5000	0.01414214	0.0231	200000000	
8					**Inner**
9	sum	585000000		Delta	2.5101E+12
10	sumx	15027020		Variance	3.66708545
11	sumy	9069243.48		a	0.00090873
12	sumx2	390292.986		Delta a	
13	sumxy	235401.373		b	0.56815216
14	sumy2	142000		Delta b	0.02923451
15					
16				d	0.1956716
17				Delta d	0.01090655
18					
19	Voltage	1/ SQRT(V)	Diameter	W(i)	
20	2500	0.02	0.055	25000000	
21	3000	0.01825742	0.0519	43200000	
22	3500	0.01690309	0.0487	68600000	
23	4000	0.01581139	0.0446	102400000	
24	4500	0.01490712	0.043	145800000	
25	5000	0.01414214	0.0404	200000000	
26					**Outer**
27	sum	585000000		Delta	5.8385E+12
28	sumx	25874340		Variance	4.4414109
29	sumy	9069243.48		a	-0.0009553
30	sumx2	1154393.07		Delta a	
31	sumxy	404843.154		b	0.37211007
32	sumy2	142000		Delta b	0.02109544
33					
34				d	0.12815471
35				Delta d	0.00776696

Fit to a Straight Line

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1. Introduction

The following describes a procedure for fitting a straight line to data containing error bars. An example is shown below where a straight line (y = a + bx) is fit to a set of data $\{x_i, y_i\}$. The purpose of the fit is to determine the following quantities:

a = the "y" intercept,
δa = the uncertainty in the "y" intercept,
b = the slope, and
δb = the uncertainty in the slope.

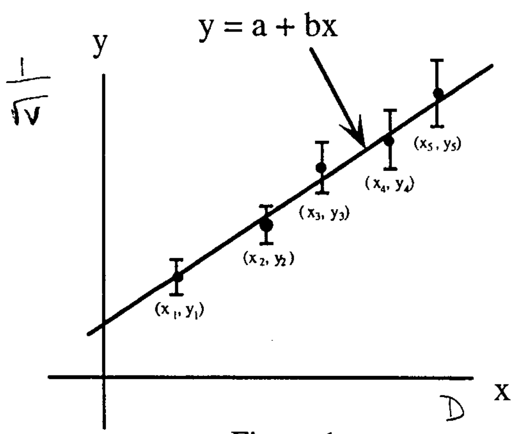

Figure 1

2. Calculations

The following sums must be calculated before calculated the intercept *(a)* and the slope *(b)*:

$$S = \sum_{i=1}^{N} w_i \qquad S_{xx} = \sum_{i=1}^{N} w_i x_i^2$$

$$S_x = \sum_{i=1}^{N} w_i x_i \qquad S_{xy} = \sum_{i=1}^{N} w_i x_i y_i$$

$$S_y = \sum_{i=1}^{N} w_i y_i \qquad S_{yy} = \sum_{i=1}^{N} w_i y_i^2$$

where N is the number of data points (e.g., N = 5 in figure 1) and w_i is the *weight* of each data point

$$w_i = \frac{1}{\delta_i^2} \ .$$

δ_i is the distance from the data point $\{x_i, y_i\}$ to the top (or bottom) of the error bar.

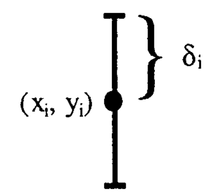

Figure 2

Next, the determinant *(Δ)* must be calculated.

$$\Delta = \begin{vmatrix} S & S_x \\ S_x & S_{xx} \end{vmatrix} = S \cdot S_{xx} - (S_x)^2$$

Now the intercept and slope *(a* and *b)* can be calculated.

$$a = \frac{\begin{vmatrix} S_{xx} & S_x \\ S_{xy} & S_y \end{vmatrix}}{\Delta} \qquad b = \frac{\begin{vmatrix} S_{xy} & S_x \\ S_y & S \end{vmatrix}}{\Delta} \ .$$

Once the slope and intercept are calculated, then the variance (σ^2) can be calculated.

$$\sigma^2 = \frac{\left(S_{yy} + a^2 S + b^2 S_{xx} - 2\left(aS_y + bS_{xy} - abS_x\right)\right)}{N-2}$$

where *N-2* is the number of degrees of freedom (e.g., in figure 1, N-2 = 3).

3. Uncertainties in *a* and *b*

Finally, the uncertainties in a and b can be determined (i.e., δa and δb) by using the previous calculations.

$$\delta a = \sqrt{\frac{\sigma^2 S_{xx}}{\Delta}} \qquad \delta b = \sqrt{\frac{\sigma^2 S}{\Delta}}$$

The final answers (including their uncertainties) can now be written as

intercept = $a \pm \delta a$ and

slope = $b \pm \delta b$.